Testing New Conditioned Balance Equations by Simulating Turbulent Scalar Transport in Impinging Jet Flames

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This work was supported
by the Swedish Research Council

Conditioned Balance Equations

Combust. Flame **152** (2008) 529-547

$$\frac{\partial}{\partial t}\left(\bar{\rho}\tilde{c}\right)+\frac{\partial}{\partial x_k}\left(\bar{\rho}\tilde{c}\bar{u}_{kb}\right)=\boxed{\gamma\overline{\left(\rho\frac{Dc}{Dt}\right)}_f}$$

$$\frac{\partial}{\partial t}\left[\bar{\rho}\tilde{c}\bar{u}_{ib}\right]+\frac{\partial}{\partial x_k}\left[\bar{\rho}\tilde{c}\bar{u}_{ib}\bar{u}_{kb}\right]=-\boxed{\frac{\partial}{\partial x_k}\left[\bar{\rho}\tilde{c}\left(\overline{u_i'u_k'}\right)_b\right]}-\boxed{\bar{c}\overline{\left(\frac{\partial p}{\partial x_i}\right)}_b}+\boxed{\bar{c}\overline{\left(\frac{\partial \tau_{ik}}{\partial x_k}\right)}_b}+\boxed{\gamma\overline{\left\{\rho\frac{D}{Dt}\left[u_i c\right]\right\}}_f}$$

$$\frac{\partial}{\partial t}\left[\bar{\rho}\tilde{c}\left(\overline{u_i'u_j'}\right)_b\right]+\frac{\partial}{\partial x_k}\left[\bar{\rho}\tilde{c}\bar{u}_{kb}\left(\overline{u_i'u_j'}\right)_b\right]+\bar{\rho}\tilde{c}\left(\overline{u_i'u_k'}\right)_b\frac{\partial\bar{u}_{jb}}{\partial x_i}+\bar{\rho}\tilde{c}\left(\overline{u_j'u_k'}\right)_b\frac{\partial\bar{u}_{ib}}{\partial x_i}$$

$$=-\boxed{\frac{\partial}{\partial x_k}\left[\bar{\rho}\tilde{c}\left(\overline{u_i'u_j'u_k'}\right)_b\right]}-\boxed{\bar{c}\overline{\left(u_j'\frac{\partial p}{\partial x_i}\right)}_b-\bar{c}\overline{\left(u_i'\frac{\partial p}{\partial x_j}\right)}_b}+\boxed{\bar{c}\overline{\left(u_j'\frac{\partial \tau_{ik}}{\partial x_k}\right)}_b+\bar{c}\overline{\left(u_i'\frac{\partial \tau_{jk}}{\partial x_k}\right)}_b}$$

$$+\boxed{\gamma\overline{\left\{\rho\frac{D}{Dt}\left[u_iu_jc\right]\right\}}_f-\gamma\bar{u}_{jb}\overline{\left\{\rho\frac{D}{Dt}\left[u_ic\right]\right\}}_f-\gamma\bar{u}_{ib}\overline{\left\{\rho\frac{D}{Dt}\left[u_jc\right]\right\}}_f+\gamma\bar{u}_{ib}\bar{u}_{jb}\overline{\left\{\rho\frac{Dc}{Dt}\right\}}_f}$$

Flamelet Terms: A Key Issue

$$\gamma\overline{\left\{\rho\frac{Dc}{Dt}\right\}_f} = \bar{\rho}\tilde{w}$$

$$\gamma\bar{u}_{jb}\overline{\left\{\rho\frac{D}{Dt}\left[u_i c\right]\right\}_f} \qquad \gamma\overline{\left\{\rho\frac{D}{Dt}\left[u_i u_j c\right]\right\}_f}$$

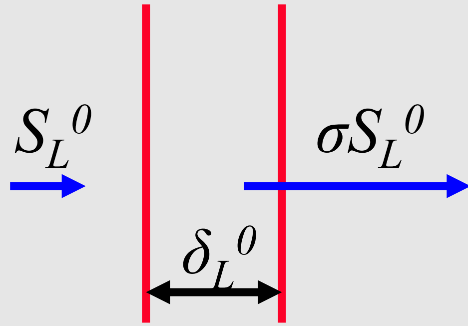
addressed by many models

new terms

If the new terms may be closed using a submodel for the mean reaction rate, then the conditioned balance equations substantially facilitate modeling of the effects of a premixed flame on turbulent flow.

The Simplest Closure

If flamelet structure is assumed to be unperturbed by turbulent eddies, then



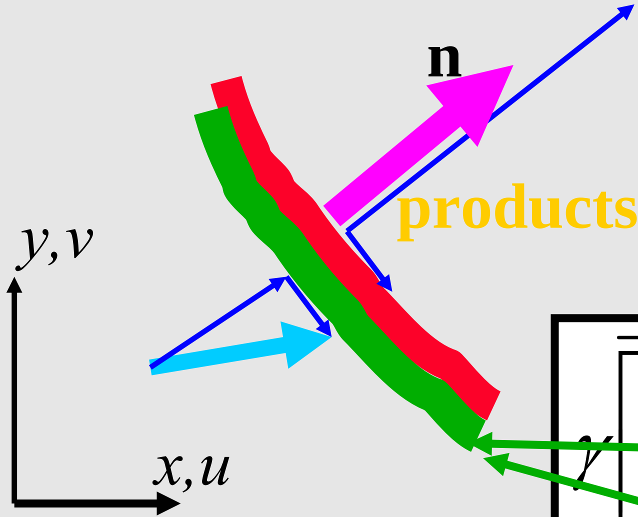
$$\overline{\left(\rho\frac{Dq}{Dt}\right)}_f = \rho_u S_L^0 \int_\varepsilon^{1-\varepsilon} \frac{dq}{dx} P_f(c)\,dc = \rho_u S_L^0 \int_\varepsilon^{1-\varepsilon} \frac{dq}{dx}\left(\frac{dc}{dx}\right)^{-1}\frac{dc}{\delta_L^0} = \frac{\rho_u S_L^0}{\delta_L^0}\left(q_b - q_u\right)$$

$$\left.\begin{aligned}
\bar{\rho}\tilde{w} = \gamma\overline{\left(\rho\frac{Dc}{Dt}\right)}_f &= \gamma\frac{\rho_u S_L^0}{\delta_L^0} \\
\gamma\overline{\left[\rho\frac{D}{Dt}(uc)\right]}_f &= \gamma\frac{\rho_u S_L^0}{\delta_L^0}\sigma S_L^0 \\
\gamma\overline{\left[\rho\frac{D}{Dt}\left(u^2 c\right)\right]}_f &= \gamma\frac{\rho_u S_L^0}{\delta_L^0}\left(\sigma S_L^0\right)^2
\end{aligned}\right\} \Rightarrow$$

$$\boxed{\gamma\overline{\left[\rho\frac{D}{Dt}(uc)\right]}_f = \sigma S_L^0\,\bar{\rho}\tilde{w}}$$

$$\boxed{\gamma\overline{\left[\rho\frac{D}{Dt}\left(u^2 c\right)\right]}_f = \left(\sigma S_L^0\right)^2\bar{\rho}\tilde{w}}$$

Three-Dimensional Closure



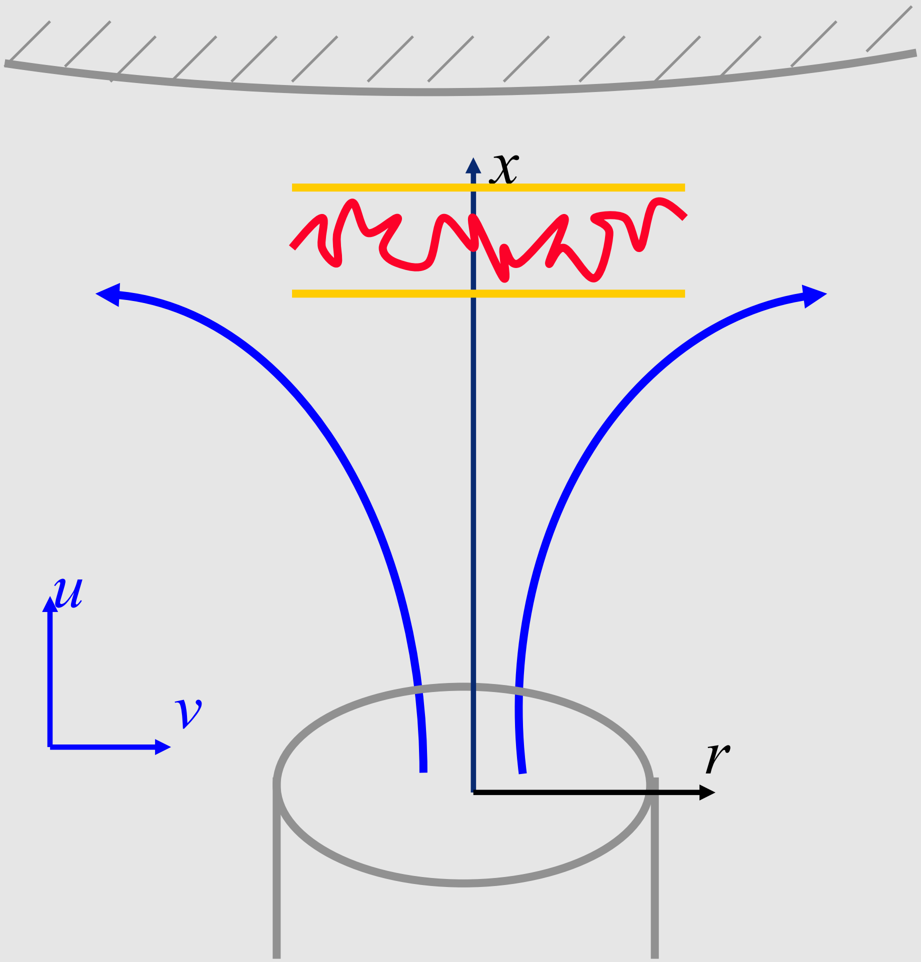
$$\gamma\overline{\left[\rho\frac{D}{Dt}\left(u_i c\right)\right]_f} = \bar{\rho}\tilde{w}\left[\left(\sigma-1\right)S_L\bar{n}_i + \bar{u}_{i,f,u}\right]$$

$$\gamma\overline{\left[\rho\frac{D}{Dt}\left(u_i u_j c\right)\right]_f} = \bar{\rho}\tilde{w}\left[\overline{n_i n_j}\delta_{i,j}\left(\sigma-1\right)^2 S_L^2 + \left(\overline{n_i u_{j,f,u}} + \overline{n_j u_{i,f,u}}\right)\left(\sigma-1\right)S_L + \left(\overline{u_i u_j}\right)_{f,u}\right]$$

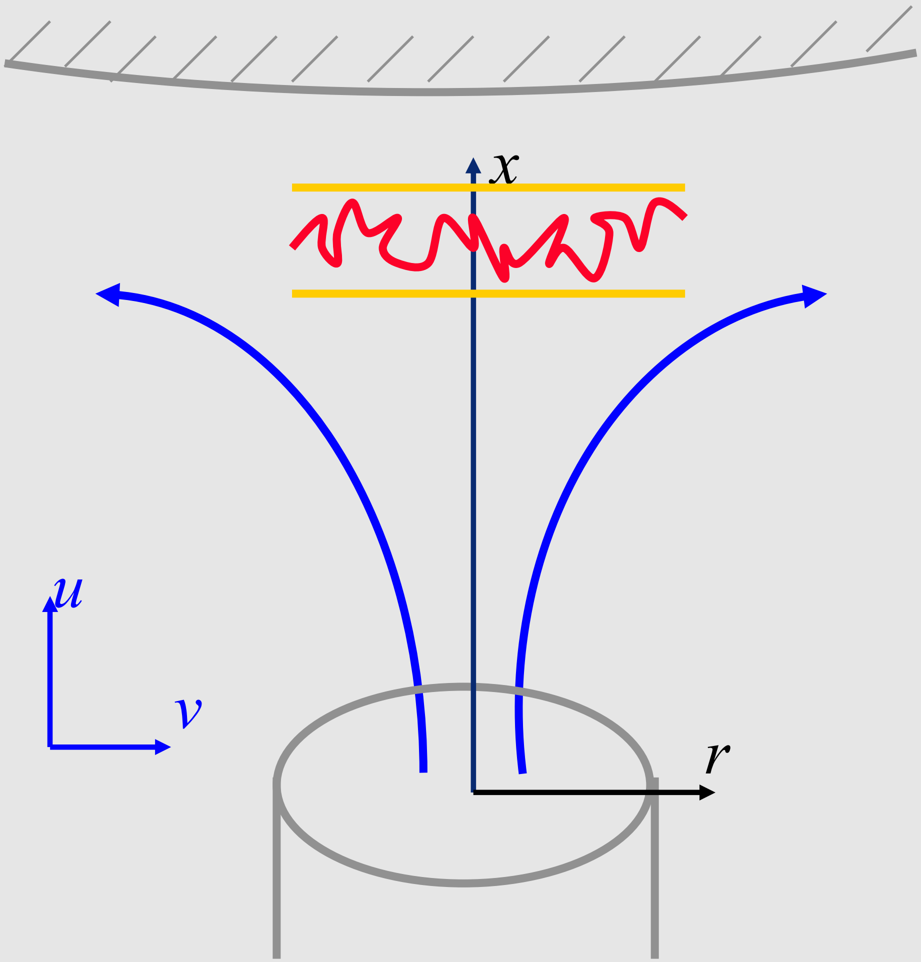
The goal of the present work is

- *to perform the first test of the conditioned balance equations and*
- *to show that they are able to facilitate modeling the effects of premixed flame on turbulent flow.*

Premixed Turbulent Stagnation Flame Stabilized in an Impinging Jet

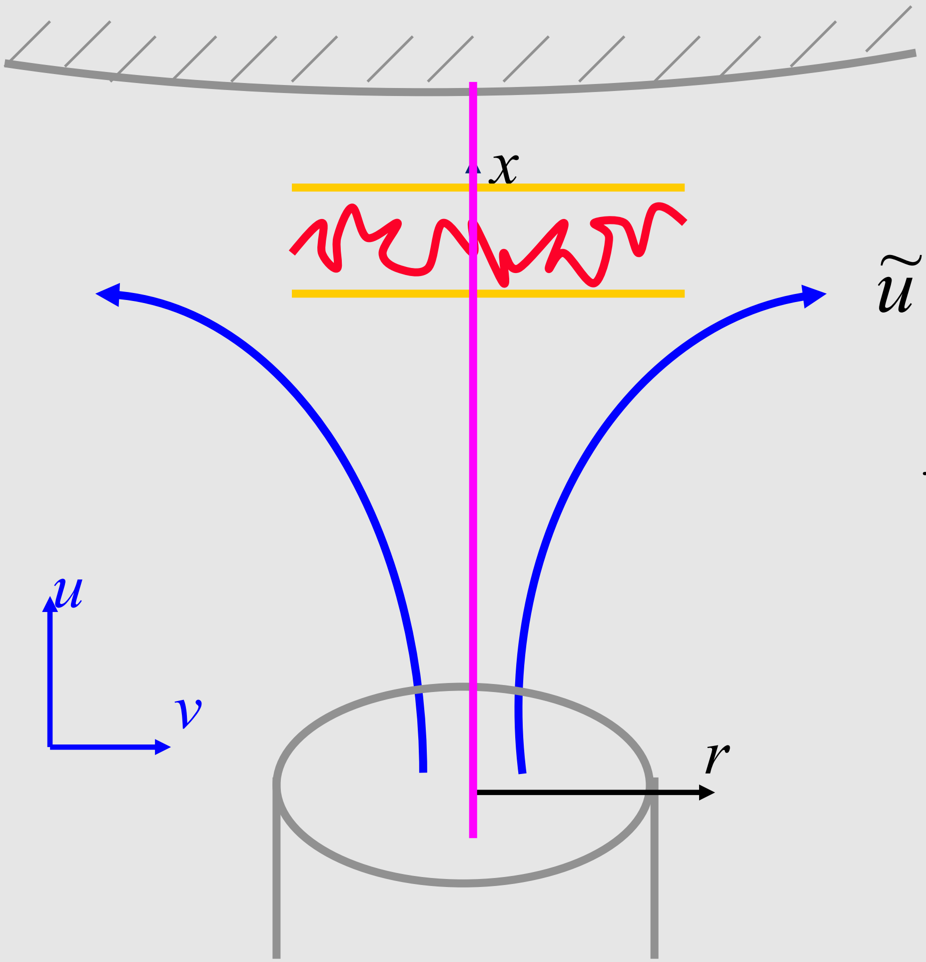


Why Stagnation Flames?



1. **Strong effect of combustion on scalar transport**

Why Stagnation Flames?



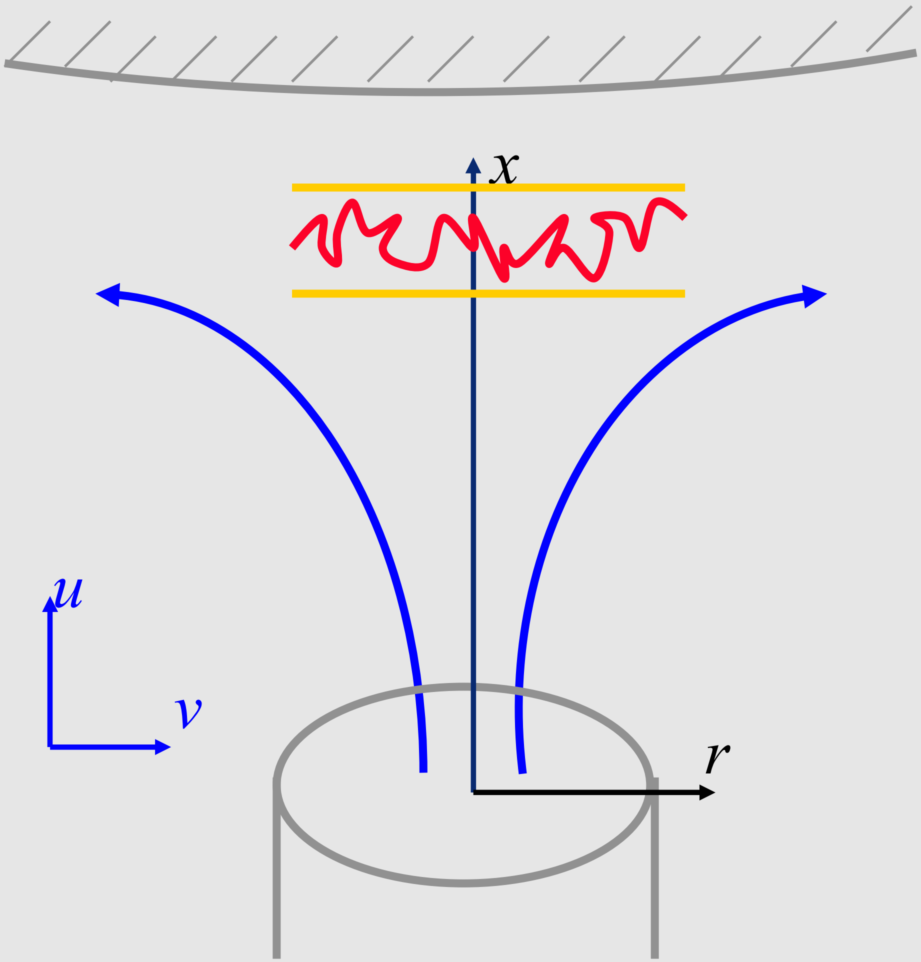
1. **Strong effect of combustion on scalar transport**
2. **Quick simulations**

$$\tilde{u}(x,r) \approx \tilde{u}(x) \qquad \tilde{c}(x,r) \approx \tilde{c}(x)$$

$$\frac{\partial \tilde{v}}{\partial r}(x,0) = 0 \qquad \tilde{v}(x,r) \approx r\tilde{g}(x)$$

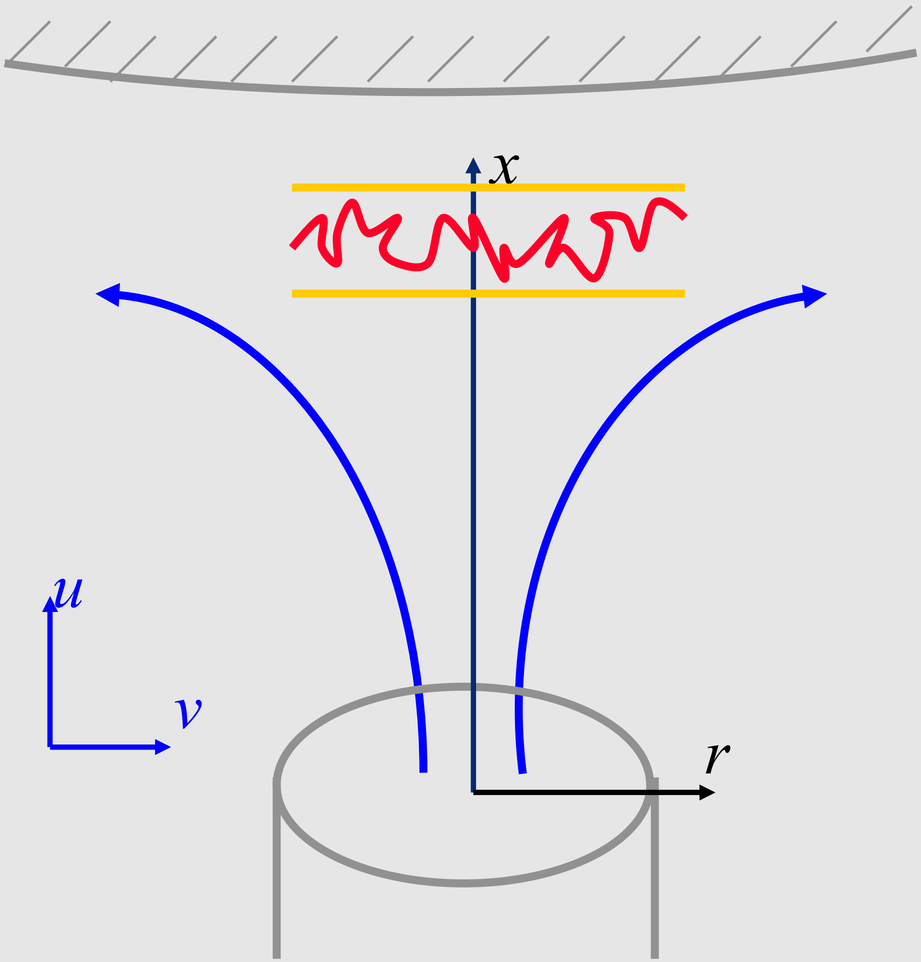
$$\boxed{\begin{aligned} &\frac{d}{dx}\left(\overline{\rho}\tilde{u}\right) + 2\overline{\rho}\tilde{g} = 0; \\ &\frac{d}{dx}\left(\overline{\rho}\tilde{u}\tilde{g}\right) + 3\overline{\rho}\tilde{g}^2 = \frac{Q}{\rho_u U_1^2} \end{aligned}}$$

Why Stagnation Flames?



1. **Strong effect of combustion on scalar transport**
2. **Quick simulations**
3. **Target-directed, "pure" test**

Why Stagnation Flames?



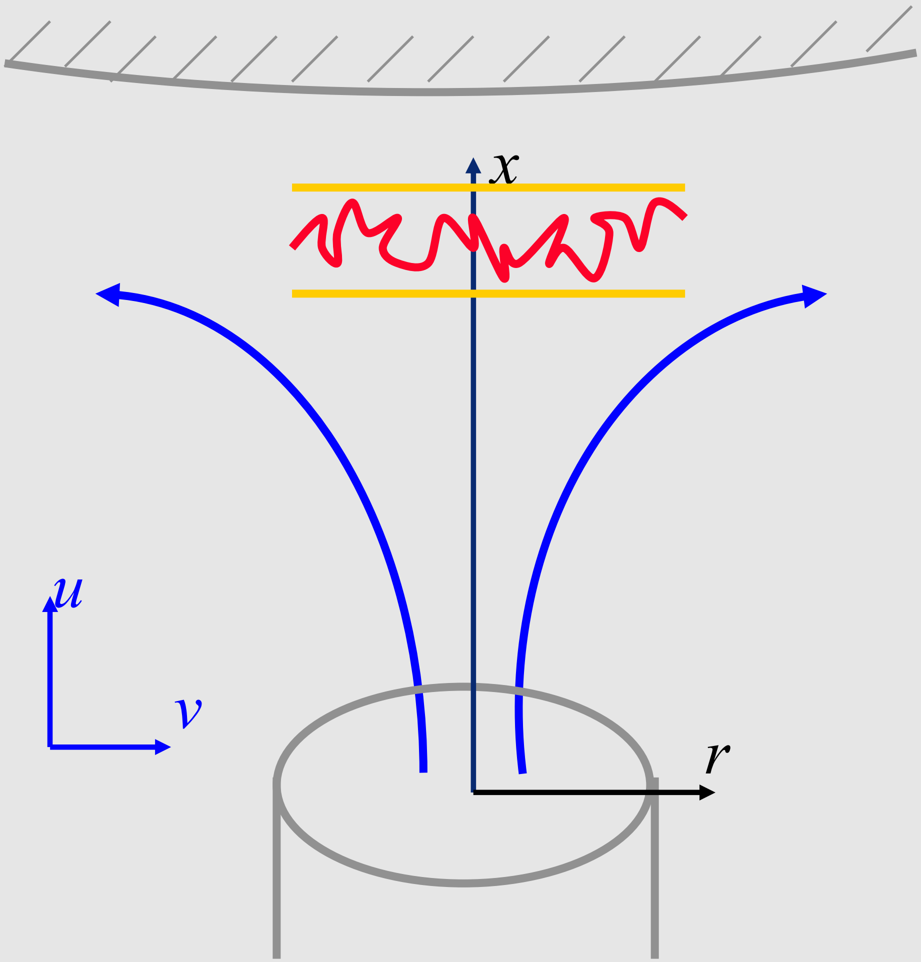
1. **Strong effect of combustion on scalar transport**
2. **Quick simulations**
3. **Target-directed, "pure" test**
4. **Representative test**

- *Cho et al. (1988)*
- *Cheng & Shepherd (1991)*
- *Li et al. (1994)*
- *Stevens et al. (1998)*

Conditions of Measurements and Simulations

d m	U_1 m/s	Fuel	Φ	S_L m/s	σ	u'/S_L	Flame
0.075	5	CH_4	1.0	0.37	7.51	1.23	#1 Cho et al.
0.1	5	C_2H_6	1.0	0.76	8.00	0.53	S9 Cheng & Shepherd
0.03	3.6	CH_4	0.89	0.31	7.08	0.70	h4 Li et al.
0.03	3.6	CH_4	0.89	0.31	7.08	0.70	h6 Li et al.
0.035	0.75	CH_4	0.6	0.085	5.54	0.71	set 1 Stevens et al.
0.035	3	CH_4	1.0	0.37	7.51	0.90	set 2 Stevens et al.
0.035	2.25	CH_4	1.3	0.21	7.11	0.85	set 3 Stevens et al.

Why Stagnation Flames?



1. Strong effect of combustion on scalar transport
2. Quick simulations
3. Target-directed, “pure” test
4. Representative test
5. Challenge

- *Cheng & Shepherd (1991)*
- *Li et al. (1994)*
- *Stevens et al. (1998)*
- *Cho et al. (1988)*

Main Goal of Simulations

To validate the conditioned balance equations

$$\frac{\partial}{\partial t}\left[\bar{\rho}\left(1-\tilde{c}\right)\right]+\frac{\partial}{\partial x_k}\left[\bar{\rho}\left(1-\tilde{c}\right)\bar{u}_{ku}\right]=-\gamma\overline{\left(\rho\frac{Dc}{Dt}\right)}_f$$

$$\frac{\partial}{\partial t}\left(\bar{\rho}\tilde{c}\right)+\frac{\partial}{\partial x_k}\left(\bar{\rho}\tilde{c}\bar{u}_{kb}\right)=\gamma\overline{\left(\rho\frac{Dc}{Dt}\right)}_f$$

$$\frac{\partial}{\partial t}\left[\bar{\rho}\left(1-\tilde{c}\right)\bar{u}_{iu}\right]+\frac{\partial}{\partial x_k}\left[\bar{\rho}\left(1-\tilde{c}\right)\bar{u}_{iu}\bar{u}_{ku}\right]=-\cancel{\frac{\partial}{\partial x_k}\left[\bar{\rho}\left(1-\tilde{c}\right)\overline{\left(u_i'u_k'\right)}_u\right]}-\left(1-\bar{c}\right)\overline{\left(\frac{\partial p}{\partial x_i}\right)}_u+\cancel{\left(1-\bar{c}\right)\overline{\left(\frac{\partial \tau_{ik}}{\partial x_k}\right)}_u}+\gamma\overline{\left\{\rho\frac{D}{Dt}\left[u_i\left(1-c\right)\right]\right\}}_f$$

$$\frac{\partial}{\partial t}\left[\bar{\rho}\tilde{c}\bar{u}_{ib}\right]+\frac{\partial}{\partial x_k}\left[\bar{\rho}\tilde{c}\bar{u}_{ib}\bar{u}_{kb}\right]=-\cancel{\frac{\partial}{\partial x_k}\left[\bar{\rho}\tilde{c}\overline{\left(u_i'u_k'\right)}_b\right]}-\bar{c}\overline{\left(\frac{\partial p}{\partial x_i}\right)}_b+\bar{c}\cancel{\overline{\left(\frac{\partial \tau_{ik}}{\partial x_k}\right)}_b}+\gamma\overline{\left\{\rho\frac{D}{Dt}\left[u_i c\right]\right\}}_f$$

supplemented with the simplest closure of flamelet terms

$$\gamma\overline{\left[\rho\frac{Du_i}{Dt}\right]}_f=\bar{\rho}\tilde{w}\left(\sigma-1\right)S_L\bar{n}$$

$$\gamma\overline{\left[\rho\frac{D}{Dt}\left(u_i c\right)\right]}_f=\bar{\rho}\tilde{w}\left[\left(\sigma-1\right)S_L\bar{n}_i+\bar{u}_{i,f,u}\right]$$

by simulating countergradient scalar transport

Key Equation

$$\frac{1}{2}\frac{d}{d\zeta}\left(\overline{u}_b^2-\overline{u}_u^2\right)=-\sigma\overline{\rho}\underbrace{\left[\overline{\left(\frac{dp}{d\zeta}\right)_b}-\overline{\left(\frac{dp}{d\zeta}\right)_u}\right]}_{\text{turbulent diffusion}}-(\sigma-1)\frac{d\overline{p}}{d\zeta}$$

$$+(\sigma-1)S_L\overline{n}_x\frac{\Omega}{\widetilde{c}}-\left(\sigma\frac{\overline{u}_b-\overline{u}_{f,u}}{\overline{c}}-\frac{\overline{u}_u-\overline{u}_{f,u}}{1-\overline{c}}\right)\Omega$$

$$\zeta\equiv\frac{x}{d}$$

$$\boxed{\overline{\rho u''c''}\approx\overline{\rho}\widetilde{c}\left(1-\widetilde{c}\right)\left(u_b-u_u\right)}$$

$$\Omega\equiv\frac{d}{\rho_u U_1}\overline{\rho}\widetilde{w}$$

Closure I: Difference in Conditioned Pressure Gradients

$$\boxed{P(t,\vec{x},q)=(1-\overline{c})\delta(q-q_u)+\gamma(t,\vec{x})P_f(t,\vec{x},q)+\overline{c}\,\delta(q-q_b)}$$

$$\Downarrow$$

$$\boxed{\overline{c'\nabla p'}-\gamma\left(\overline{c'\nabla p'}\right)_f=\overline{c}(1-\overline{c})\underbrace{\left[\left(\overline{\nabla p}\right)_b-\left(\overline{\nabla p}\right)_u\right]}_{\text{non-reacting terms}}}$$

Launder's closure

Inert constant-density flows:

$$\overline{c}(1-\overline{c})\left[\left(\overline{\nabla p}\right)_b-\left(\overline{\nabla p}\right)_u\right]=\overline{c'\nabla p'}=\boxed{-C_1\frac{\overline{\varepsilon}}{\overline{k}}\overline{\mathbf{u}'c'}}=\boxed{C_2\overline{k}_u\nabla\overline{c}}$$

gradient diffusion closure

$$\boxed{\overline{c}(1-\overline{c})\left[\left(\overline{\nabla p}\right)_b-\left(\overline{\nabla p}\right)_u\right]=C_2\overline{\rho}\overline{k}_u\nabla\widetilde{c}}\qquad C_2\approx 1/2$$

Key Equation

$$\frac{1}{2}\frac{d}{d\zeta}\left(\bar{u}_b^2-\bar{u}_u^2\right)=-\sigma\bar{\rho}\overbrace{\left[\overline{\left(\frac{dp}{d\zeta}\right)_b}-\overline{\left(\frac{dp}{d\zeta}\right)_u}\right]}^{\text{turbulent diffusion}}\overbrace{-(\sigma-1)\frac{d\bar{p}}{d\zeta}}^{\text{mean pressure gradient}}$$

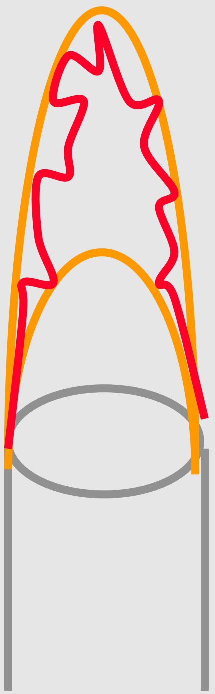
$$\underbrace{+(\sigma-1)S_L\bar{n}_x\frac{\Omega}{\tilde{c}}-\left(\sigma\frac{\bar{u}_b-\bar{u}_{f,u}}{\bar{c}}-\frac{\bar{u}_u-\bar{u}_{f,u}}{1-\bar{c}}\right)\Omega}_{\text{flamelet terms}}$$

$$\zeta\equiv\frac{x}{d}$$

$$\boxed{\overline{\rho u''c''}\approx\bar{\rho}\tilde{c}\left(1-\tilde{c}\right)\left(u_b-u_u\right)}$$

$$\Omega\equiv\frac{d}{\rho_u U_1}\bar{\rho}\tilde{w}$$

Closure II: Flamelet Normal



Experimental data obtained by Chen & Bilger (2002) from a number of Bunsen-type premixed turbulent flames show that the projection of the averaged normal vector to flamelets on the line perpendicular to the mean flame brush is close to 2/3.

$$\boxed{\overline{n}_x = -2/3}$$

Key Equation

$$\frac{1}{2}\frac{d}{d\zeta}\left(\overline{u}_b^2-\overline{u}_u^2\right)=\underbrace{-\sigma\overline{\rho}\left[\overline{\left(\frac{dp}{d\zeta}\right)_b}-\overline{\left(\frac{dp}{d\zeta}\right)_u}\right]}_{\text{turbulent diffusion}}\underbrace{-(\sigma-1)\frac{d\overline{p}}{d\zeta}}_{\text{mean pressure gradient}}$$

$$\underbrace{+(\sigma-1)S_L\overline{n}_x\frac{\Omega}{\widetilde{c}}-\left(\sigma\frac{\overline{u}_b-\overline{u}_{f,u}}{\overline{c}}-\frac{\overline{u}_u-\overline{u}_{f,u}}{1-\overline{c}}\right)\Omega}_{\text{flamelet terms}}$$

$$\zeta\equiv\frac{x}{d}$$

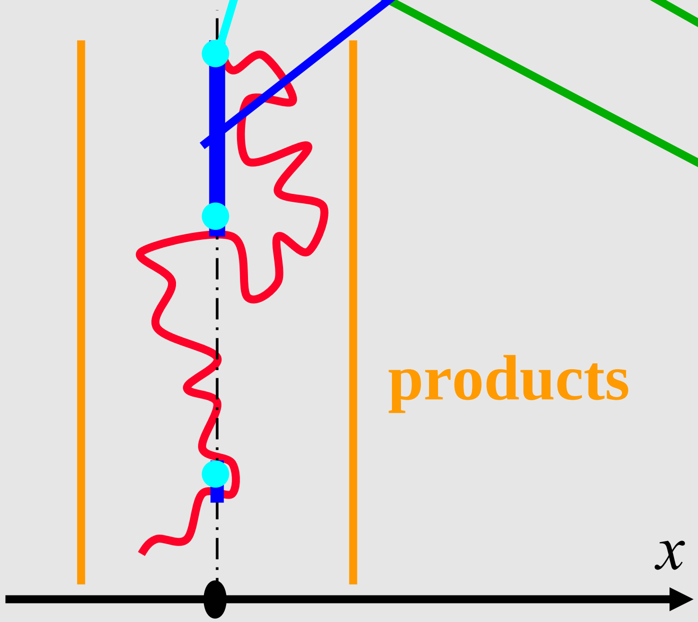
$$\boxed{\overline{\rho u''c''}\approx\overline{\rho}\widetilde{c}\left(1-\widetilde{c}\right)\left(u_b-u_u\right)}$$

$$\Omega\equiv\frac{d}{\rho_u U_1}\overline{\rho}\widetilde{w}$$

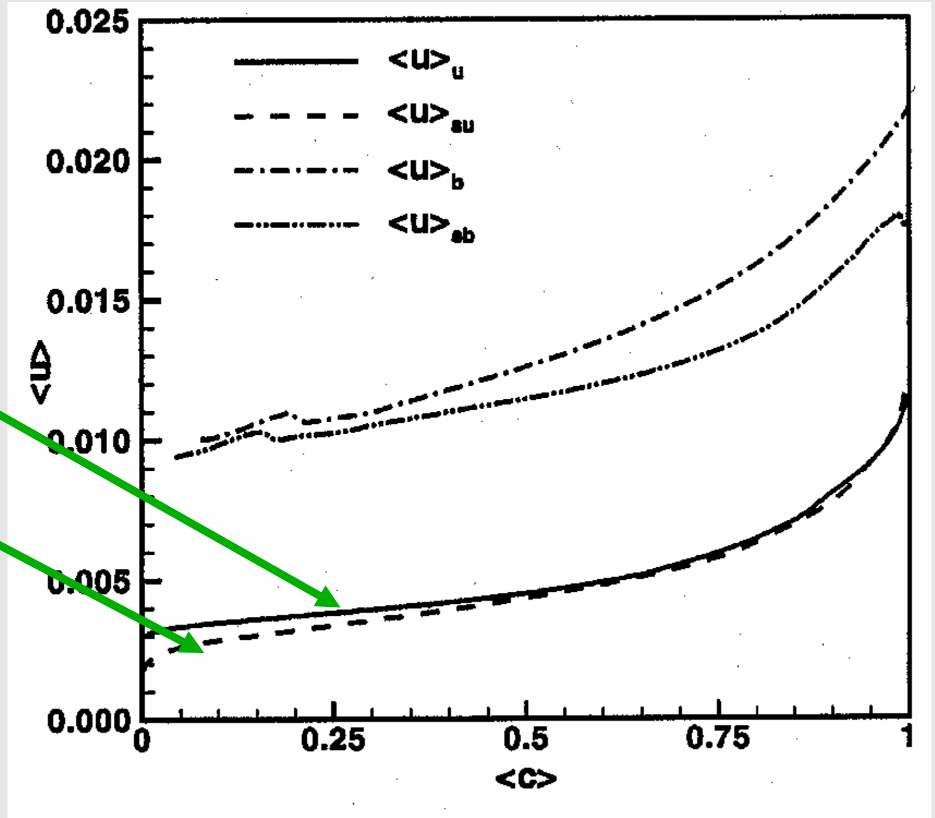
Closure III: Velocity Ahead Flamelets

The simplest closure:

$$\boxed{\overline{u}_{f,u} = \overline{u}_u}$$



DNS data by Nishiki (2004)



Key Equation

$$\frac{1}{2}\frac{d}{d\zeta}\left(\overline{u}_b^{\,2}-\overline{u}_u^{\,2}\right)=-\sigma\overline{\rho}\overbrace{\left[\overline{\left(\frac{dp}{d\zeta}\right)_b}-\overline{\left(\frac{dp}{d\zeta}\right)_u}\right]}^{\text{turbulent diffusion}}\overbrace{-(\sigma-1)\frac{d\overline{p}}{d\zeta}}^{\text{mean pressure gradient}}$$

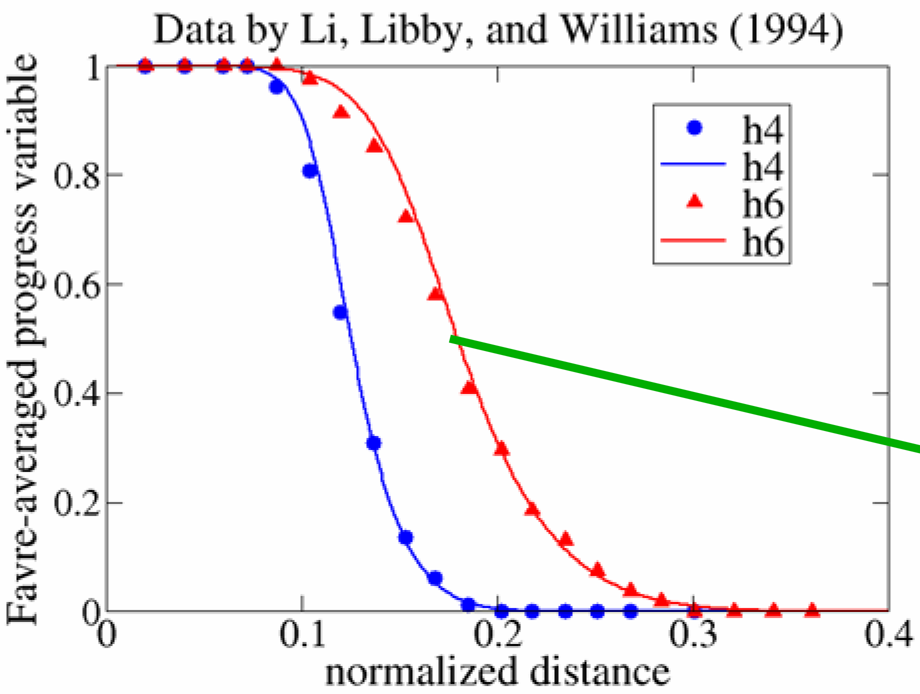
$$\underbrace{+(\sigma-1)S_L\overline{n}_x\frac{\Omega}{\tilde{c}}-\left(\sigma\frac{\overline{u}_b-\overline{u}_{f,u}}{\overline{c}}-\frac{\overline{u}_u-\overline{u}_{f,u}}{1-\overline{c}}\right)\Omega}_{\text{flamelet terms}}$$

$$\zeta\equiv\frac{x}{d}$$

$$\boxed{\overline{\rho u''c''}\approx\overline{\rho}\tilde{c}\left(1-\tilde{c}\right)\left(u_b-u_u\right)}$$

$$\Omega\equiv\frac{d}{\rho_u U_1}\overline{\rho}\tilde{w}$$

Closure IV: Mean Reaction Rate



Combustion progress variable balance equation:

$$\Omega = \frac{d}{d\zeta}\left(\bar{\rho}\tilde{u}\tilde{c}\right) + 2\bar{\rho}\tilde{g}\tilde{c} + \frac{d}{d\zeta}\left[\bar{\rho}\tilde{c}\left(1-\tilde{c}\right)\left(\bar{u}_b - \bar{u}_u\right)\right] + 2\bar{\rho}\tilde{c}\left(1-\tilde{c}\right)\left(\tilde{g}_b - \tilde{g}_u\right)$$

$$\bar{c} = \frac{1}{2}\operatorname{erfc}\left(\xi\sqrt{\pi}\right) = \frac{1}{\sqrt{\pi}}\int_{\xi\sqrt{\pi}}^{\infty} e^{-\varsigma^2}\, d\varsigma;$$

Closed Key Equations

$$\frac{1}{2}\frac{d}{d\zeta}\left(\bar{u}_b^2-\bar{u}_u^2\right)=-\cancel{\frac{3}{4}\frac{\sigma\bar{\rho}^2u'^2}{\bar{c}\left(1-\bar{c}\right)}\frac{d\tilde{c}}{d\zeta}}$$

$$-\left(\sigma-1\right)\frac{d\bar{p}}{d\zeta}-\left[\frac{2}{3}\bar{\rho}\left(\sigma-1\right)S_L-\left(\bar{u}_u-\bar{u}_b\right)\right]\frac{\sigma\Omega}{\bar{c}}$$

$$\Omega=\frac{d}{d\zeta}\left(\bar{\rho}\tilde{u}\tilde{c}\right)+\frac{d}{d\zeta}\left[\bar{\rho}\tilde{c}\left(1-\tilde{c}\right)\left(\bar{u}_b-\bar{u}_u\right)\right]$$

$$+2\bar{\rho}\tilde{g}\tilde{c}+2\bar{\rho}\tilde{c}\left(1-\tilde{c}\right)\left(\tilde{g}_b-\tilde{g}_u\right)$$

Other Equations

Mass: $$\frac{d}{d\zeta}\left(\bar{\rho}\tilde{u}\right)+2\bar{\rho}\tilde{g}=0;$$

Radial velocity: $$\frac{d}{d\zeta}\left(\bar{\rho}\tilde{u}\tilde{g}\right)+3\bar{\rho}\tilde{g}^2=Q;$$

Axial velocity: $$\frac{d}{d\zeta}\left(\bar{\rho}\tilde{u}^2\right)+\frac{d}{d\zeta}\left[\bar{\rho}\tilde{c}\left(1-\tilde{c}\right)\left(\bar{u}_b-\bar{u}_u\right)^2\right]+2\bar{\rho}\tilde{u}\tilde{g}=-\frac{d\bar{p}}{d\zeta};$$

BML state equation: $$\rho_b\bar{c}=\bar{\rho}\tilde{c}=\frac{\tilde{c}}{1+\left(\sigma-1\right)\tilde{c}}$$

Boundary conditions: $\tilde{u}(1)=-1;$ $\tilde{g}(1)=g_1;$ $\bar{u}_b(\zeta_1)=\bar{u}_u(\zeta_1)$

Parameters Q and g_1 were adjusted by comparing the measured and computed mean axial velocities within flame brush.

Target-Directed Test

Measurements

$\widetilde{c}(\zeta)$

$\widetilde{u}(\zeta)$

$$\rho_b \overline{c} = \overline{\rho}\widetilde{c} = \frac{\widetilde{c}}{1+(\sigma-1)\widetilde{c}}$$

$$\frac{1}{2}\frac{d}{d\zeta}\left(\overline{u}_b^2 - \overline{u}_u^2\right) = -\frac{3}{4}\frac{\sigma\overline{\rho}^2 u'^2}{\overline{c}\left(1-\overline{c}\right)}\frac{d\widetilde{c}}{d\zeta}$$

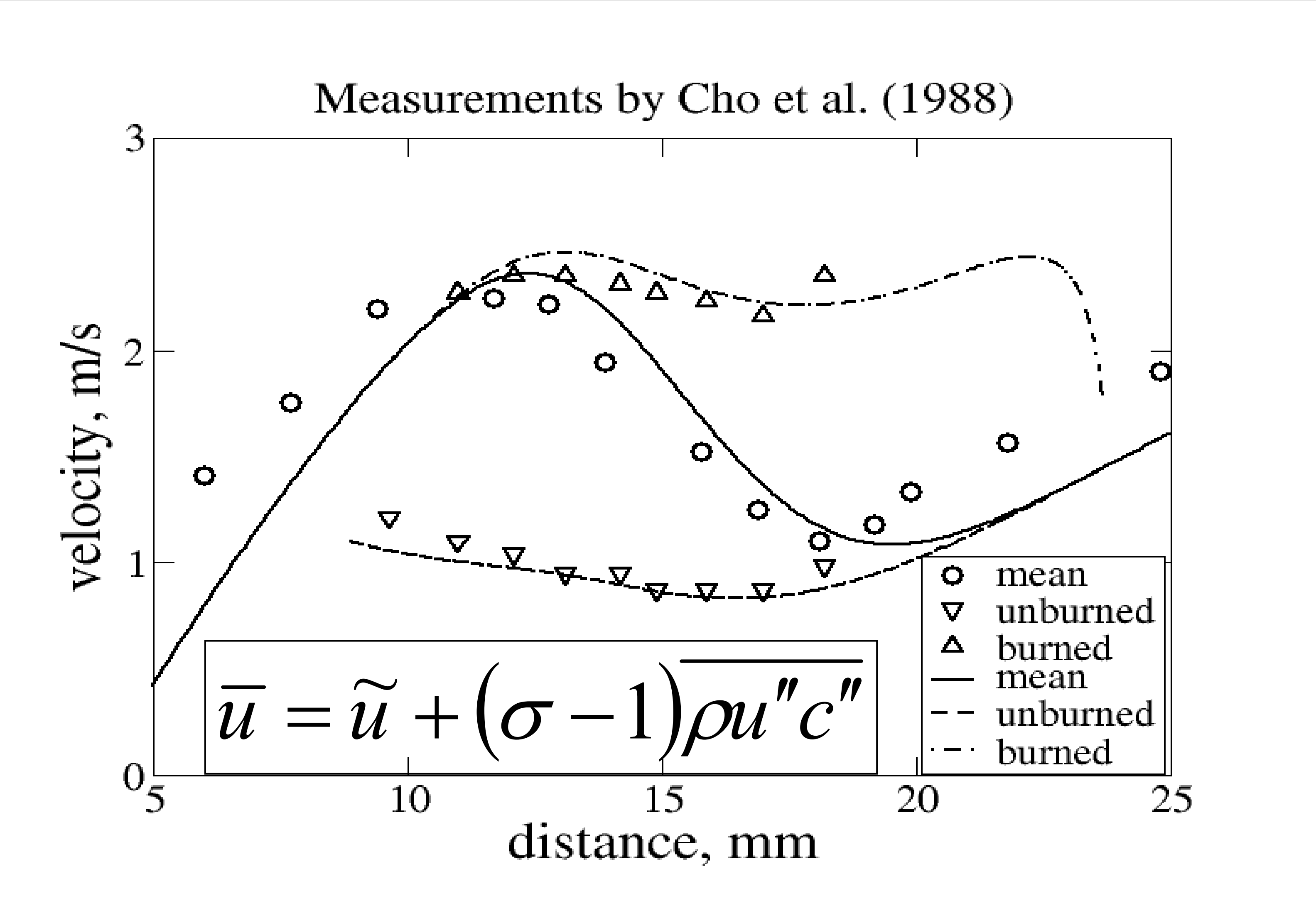
$$-(\sigma-1)\frac{d\overline{p}}{d\zeta} - \left[\frac{2}{3}\overline{\rho}(\sigma-1)S_L - \left(\overline{u}_u - \overline{u}_b\right)\right]\frac{\sigma\Omega}{\overline{c}}$$

$$\frac{d}{d\zeta}\left(\overline{\rho}\widetilde{u}^2\right) + \frac{d}{d\zeta}\left[\overline{\rho}\widetilde{c}\left(1-\widetilde{c}\right)\left(\overline{u}_b - \overline{u}_u\right)^2\right] + 2\overline{\rho}\widetilde{u}\widetilde{g} = -\frac{d\overline{p}}{d\zeta};$$

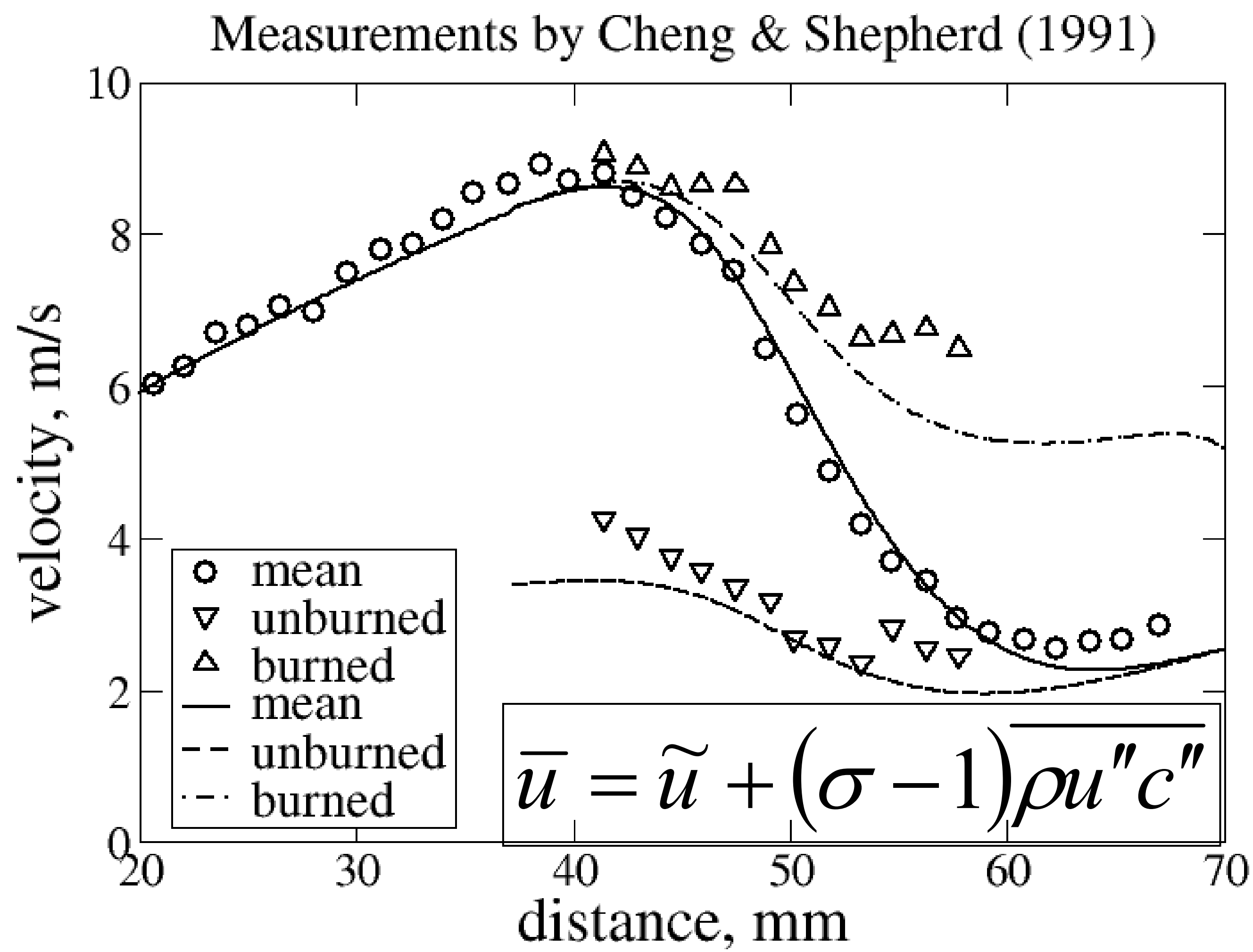
$$\Omega = \frac{d}{d\zeta}\left(\overline{\rho}\widetilde{u}\widetilde{c}\right) + \frac{d}{d\zeta}\left[\overline{\rho}\widetilde{c}\left(1-\widetilde{c}\right)\left(\overline{u}_b - \overline{u}_u\right)\right] + 2\overline{\rho}\widetilde{g}\widetilde{c} + 2\overline{\rho}\widetilde{c}\left(1-\widetilde{c}\right)\left(\widetilde{g}_b - \widetilde{g}_u\right)$$

$$\frac{d}{d\zeta}\left(\overline{\rho}\widetilde{u}\right) + 2\overline{\rho}\widetilde{g} = 0;$$

Validation I

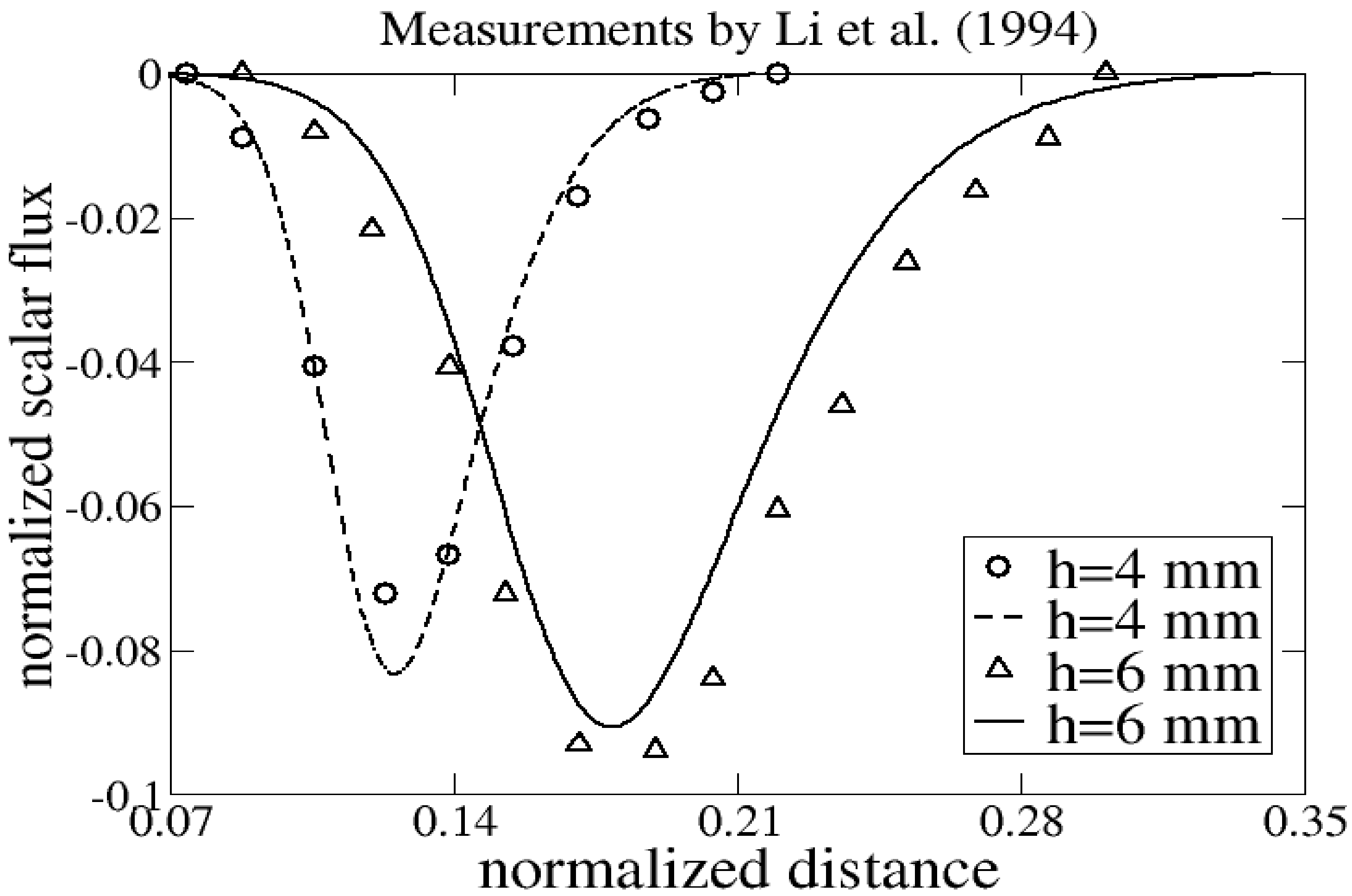


Validation II

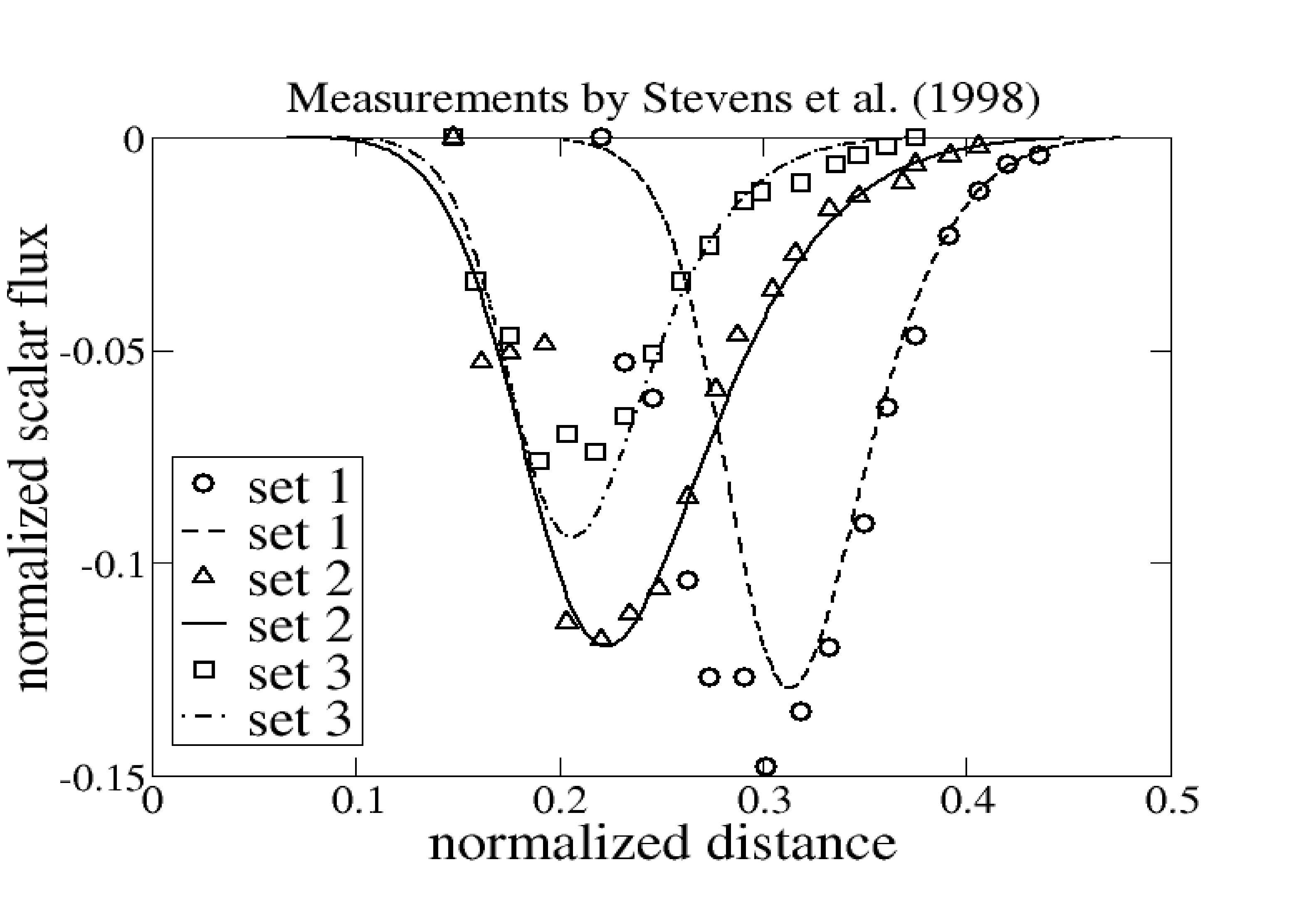


Validation III

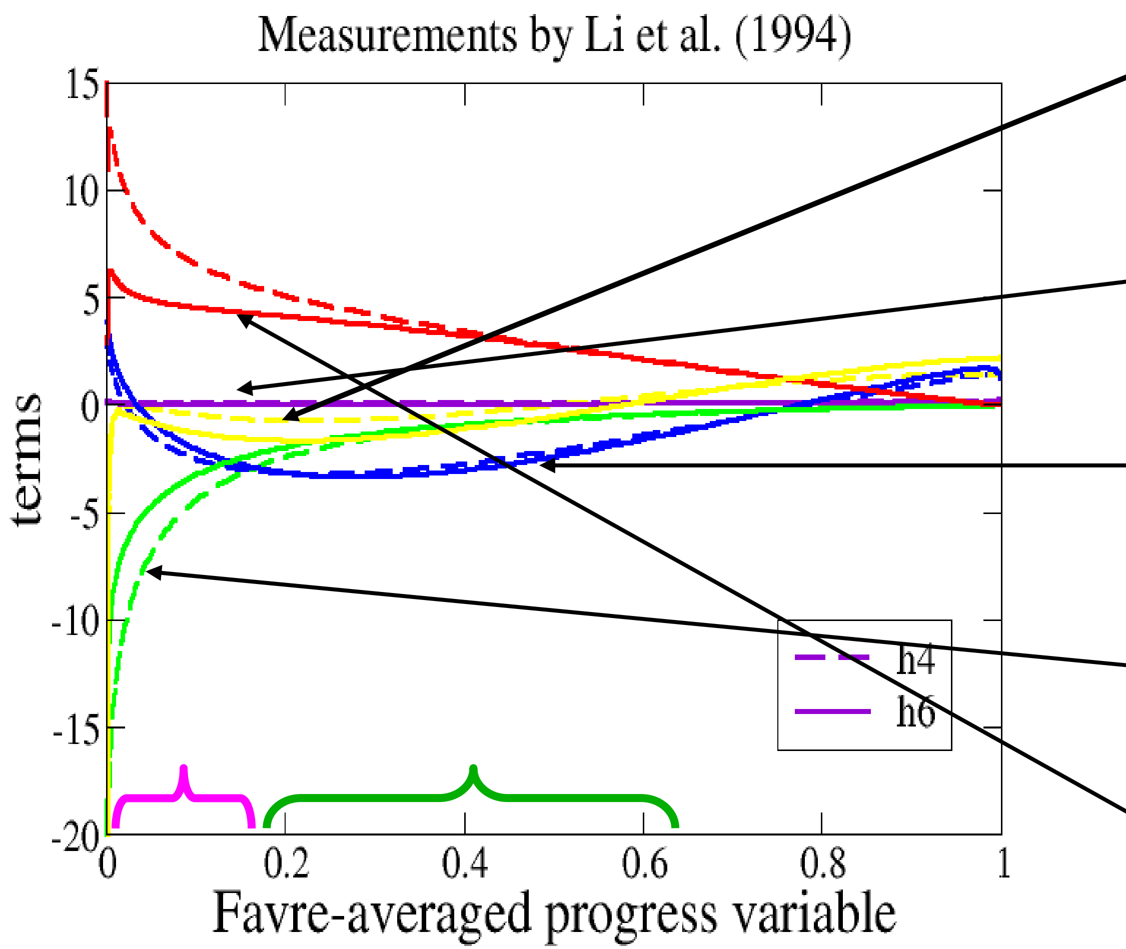
$$\frac{\overline{\rho u''c''}}{\overline{\rho}U_1}$$



Validation IV



Governing Physical Mechanisms



$$\frac{1}{2}\frac{d}{d\zeta}\left(\overline{u}_b^{\,2}-\overline{u}_u^{\,2}\right)$$

$$-\frac{3}{4}\frac{\sigma\overline{\rho}^{\,2}u'^2}{\overline{c}\left(1-\overline{c}\right)}\frac{d\tilde{c}}{d\zeta}$$

$$-\left(\sigma-1\right)\frac{d\overline{p}}{d\zeta}$$

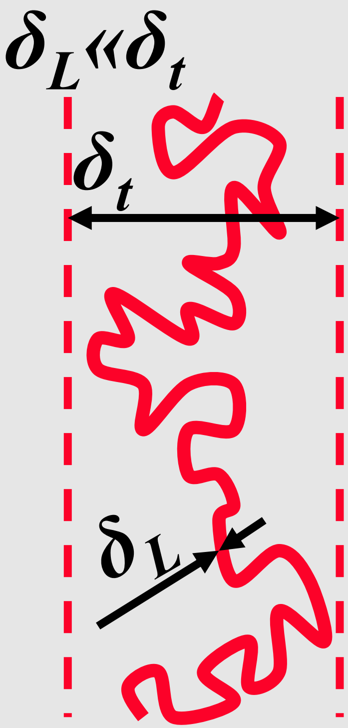
$$-\frac{2}{3}\overline{\rho}\left(\sigma-1\right)S_L\frac{\sigma\Omega}{\overline{c}}$$

$$+\left(\overline{u}_u-\overline{u}_b\right)\frac{\sigma\Omega}{\overline{c}}$$

Main Message

The proposed approach is promising!

Balance Equations for Conditioned Second Moments I



$$\overline{\rho}\widetilde{u}_i = \overline{\rho}\widetilde{c}\,\overline{u}_{iu} + \overline{\rho}\left(1-\widetilde{c}\right)\overline{u}_{ib}$$

$$\overline{\rho u_i'' c''} = \overline{\rho}\widetilde{c}\left(1-\widetilde{c}\right)\left(\overline{u}_{ib}-\overline{u}_{iu}\right)$$

$$\begin{cases} \overline{u}_{ib} = \widetilde{u}_i + \dfrac{\overline{\rho u_i'' c''}}{\overline{\rho}\widetilde{c}} \\ \overline{u}_{iu} = \widetilde{u}_i - \dfrac{\overline{\rho u_i'' c''}}{\overline{\rho}\left(1-\widetilde{c}\right)} \end{cases}$$

$$\overline{\rho u_i'' u_j''} = \overline{\rho}\left(1-\widetilde{c}\right)\left(\overline{u_i' u_j'}\right)_u + \overline{\rho}\widetilde{c}\left(\overline{u_i' u_j'}\right)_b + \overline{\rho}\widetilde{c}\left(1-\widetilde{c}\right)\left(\overline{u}_{ib}-\overline{u}_{iu}\right)\left(\overline{u}_{jb}-\overline{u}_{ju}\right)$$

$$\overline{\rho u_i'' u_j'' c''} = \overline{\rho}\widetilde{c}\left(1-\widetilde{c}\right)\left[\left(\overline{u_i' u_j'}\right)_b - \left(\overline{u_i' u_j'}\right)_u + \left(1-2\widetilde{c}\right)\left(\overline{u}_{ib}-\overline{u}_{iu}\right)\left(\overline{u}_{jb}-\overline{u}_{ju}\right)\right]$$

$$\overline{\rho}\left(1-\widetilde{c}\right)\left(\overline{u_i' u_j'}\right)_u = \left(1-\widetilde{c}\right)\overline{\rho u_i'' u_j''} - \overline{\rho u_i'' u_j'' c''} - \frac{\overline{\rho u_i'' c''}\cdot\overline{\rho u_j'' c''}}{\overline{\rho}\left(1-\widetilde{c}\right)}$$

$$\overline{\rho}\widetilde{c}\left(\overline{u_i' u_j'}\right)_b = \widetilde{c}\,\overline{\rho u_i'' u_j''} + \overline{\rho u_i'' u_j'' c''} - \frac{\overline{\rho u_i'' c''}\cdot\overline{\rho u_j'' c''}}{\overline{\rho}\widetilde{c}}$$